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EXTREME VALUES OF QUEUES POINT PROCESSES AND STOCHASTIC  
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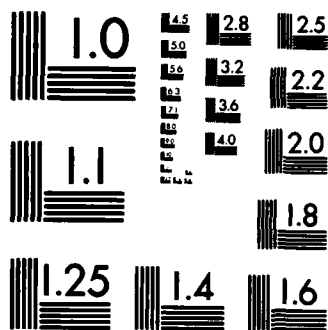
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Research Progress and Forecast Report for

"Extreme Values of Queues, Point Processes  
and Stochastic Networks"

Grant No. AFOSR 84-0367

by

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<p>Research progressed in the following topics: (1) Extreme Values of Queues: - Several subtle technical problems remain to present the results in as general and natural setting as possible; (2) Extreme Values of Stochastic Networks: - Work should be completed by fall. After that the emphasis will change on this topic to the following one; (3) Optimal Control of Networks of queues: - This topic was not in the original proposal, but the investigators have made a breakthrough in this area that they intend to pursue; (4) Point Processes Related to Extreme Values: - Several results in the convergence of certain point processes to Poisson processes have been obtained. These are documented in two papers written under this grant; (5) Extreme Values of Point Processes. - This is an area which will be developed later in the research effort.</p>					
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## 1. Summary of Activities

Our research has been evolving as follows.

- (a) Extreme Values of Queues. The work on this topic has been progressing steadily along the lines of our initial proposal. There are several subtle technical problems we have to resolve in order to present our results in as general and natural setting as possible. We expect to start documentation of this work next fall.
- (b) Extreme Values of Point Processes. Little time was spent on this topic; we will get to this later.
- (c) Extreme Values of Stochastic Networks. We are developing bounds for extreme values of specially structured networks. This should be completed by next fall. After that we are planning to change the emphasis on this topic to the following one.
- (d) Optimal Control of Networks of Queues. Although this topic was not in our original proposal, we made a breakthrough in this area that we intend to pursue. A major problem in the control of queueing and inventory systems is to establish the existence of nicely structured monotone optimal policies. This problem has been solved for several standard one-dimensional processes, but the techniques used do not extend to multi-dimensional processes such as queueing networks. However, we have developed a new approach that works for multi-dimensional processes. We plan to develop this further during the next year.
- (e) Point Processes Related to Extreme Values. The study of extreme values as in (a) and (b) is closely related to the convergence of certain point processes to Poisson processes. We have obtained some exciting results in this topic that are documented in the attached papers. Further discussion is given below.

## 2. Poisson and Compound Poisson Approximations

A standard approach for analyzing the asymptotic behavior of extreme values of a discrete-time stochastic process is via point processes. The key idea is to represent the cumulative maxima of the process as a functional of an approximate point process on the plane. One establishes the convergence of the point process to a monohomogeneous Poisson process, and then invokes the continuous mapping theorem to obtain the convergence of the maxima. This approach has been used for extreme values of independent variables and of stationary variables.

Our research on this topic was prompted by the following questions. What can one say about extremes of variables that are not independent or stationary? Is it possible that for certain dependent variables the associated point process converges to a compound Poisson or infinitely divisible point process rather than a Poisson process? What are natural conditions for this? What types of limiting distributions would the extremes have and how are they related to the three classical extreme-value distributions? Do the results hold for multi-dimensional processes? What are the rates of convergence of the extremes to their limits?

We were able to shed some light on these questions by developing several compound Poisson approximations that are of fundamental interest. The attached papers contain our results on this theme. Here is more background on them.

### Compound Poisson Approximations for Sums of Random Variables

There are a number of well-known Poisson approximations for sums of uniformly small random variables. When the random variables are generally small but not uniform, then one might expect the sum to be

approximately compound Poisson or infinitely divisible. Freedman (1975) gave examples of sums that converge to compound Poisson variables but have wild oscillations. This suggests that there may not be omnibus compound Poisson approximations analogous to Poisson or normal approximations.

This is not the case. My paper with the title above gives a compound Poisson approximation for a large class of sums of dependent variables. The results also yield theorems on the convergence of point processes to compound Poisson processes. In operations research or computer systems, most flows of goods and services or data packets are compound Poisson (batch flows) rather than Poisson. Our results should be of use in justifying the assumption of compound Poisson flows in the same way that classical results justified assumptions of Poisson flows.

Partitions of Point Processes: Multivariate Poisson Approximations

A classical result is that a sum of a large number of thin point processes is approximately Poisson. What about the reverse? When a point process is partitioned into a large number of subprocesses, are these subprocesses approximately Poisson? When might they be compound Poisson? These are the issues I address in the second paper.



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